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MOLAR MOMENTUM AND HEAT TRANSFER

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UDC 532.526

Universal relations governing the molar transfer of momentum and heat are derived on the basis of a hypothesis about the dependence of the boundaries of the molar transfer region on the flow structure and with the use of a special mathematical transformation.

Molar transfer, i.e., the transfer of momentum, heat, mass, and other entities by finite masses of a continuum, is commonplace in nature and technology. The well-known molar transfer relations contain empirical constants and are not universal [1].

We now attempt to establish universal relations for steady axisymmetrical and plane molar momentum- and heat-transfer processes in the turbulent core of a turbulent boundary layer.

It has been shown [2] that the following generalized relation holds for molar momentum transfer in a turbulent boundary layer with zero pressure gradient:

$$\frac{d\tilde{U}}{d\tilde{R}} = 1, \tag{1}$$

where $\tilde{U} = (u^+ - 1)/(u^+_\delta - 1)$, $\tilde{R} = \ln y^+/\ln \delta^+$.

Supervisory Council, State Committee of the USSR on Inventions and Discoveries. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 3, pp. 441-448, September, 1981. Original article submitted June 23, 1980.

 0022-0841/81/4103-0960$07.50

Equation (1) has been obtained on the basis of the hypothesis of quantization of the turbulent boundary layer into average moles (eddies), where the domain of definition of the average longitudinal velocity u(x, y) is given as follows:

$$x_2 \geqslant x \geqslant x_1 \geqslant 0, \quad \delta(x) \geqslant y \geqslant l_*(x) > 0. \tag{2}$$

Here x is the longitudinal coordinate measured along the bounding solid surface (wall), y is the transverse coordinate measured from the wall, $l_*(x)$ is a transverse space scale of the average mole (eddy) formed at the wall, and $\delta(x)$ is the boundary-layer thickness.

It has also been proved that if the average longitudinal velocity u(x, y) with the domain (2) is transformed to the dimensionless velocity $\tilde{U}$, which is a single-valued function of one generalized variable $\tilde{R}(x, y)$, for the purpose of obtaining a universal relation for $\tilde{U}(\tilde{R})$ in the interval $[\tilde{R}_1, \tilde{R}_2]$, expression (1) is valid. In this case $\tilde{R}_2 = 1$, and $R_1 \geqslant \ln \delta_0^+/\ln \delta^+$. ($\tilde{R}_1 = \ln \delta_0^+/\ln \delta^+$ if the turbulent boundary layer does not contain a Kármán transition zone.) Here δ_0^+ is the dimensionless thickness of the laminar sublayer.

We assume that the boundaries of the molar transfer domain in, for example, the turbulent core of a turbulent boundary layer depend on the structure of the flow; we specify this domain in the form

$$x_2 \geqslant x \geqslant x_1 \geqslant 0, \quad \delta(x) \geqslant y \geqslant \delta_0(x) > 0. \tag{3}$$

We speak of the boundary curve $\delta(x)$ as the upper boundary of the molar transfer domain, for example the thickness of the turbulent boundary layer, and we interpret $\delta_0(x)$ as the lower boundary, in particular the thickness of the laminar sublayer.

The domain (3) is far more general than (2); for example, the representation (3) obviates the need for additional knowledge of the structure of (2) and, hence, for specifying the scale of the average mole at the wall l_*.

The universality of the longitudinal-velocity and temperature profiles in the molecular transfer domain, for example in the laminar sublayer of a turbulent boundary layer, is governed by the universality in that domain of Newton's law of molecular momentum transfer

$$\frac{du^+}{dy^+} = 1 \tag{4}$$

and the universality of the Fourier law of molecular heat transfer

$$\frac{d\vartheta^+}{d(\Pr y^+)} = 1. \tag{5}$$

If the profiles of the average longitudinal velocity u(x, y) and the average thermodynamic temperature T(x, y) in the molar transfer zone are also universal, then they must be described by their own universal laws, but now for the molar transfer of momentum and heat.

The indicated laws will be the only possible generalized relations for u(x, y) and T(x, y) in the domain (3).

Using the previously described [2] special transformation of a function of two variables into a generalized single-valued function of one generalized variable, we obtain generalized relations for the molar momentum- and heat-transfer process under the condition $(du^+/dy^+)_0 > 0$:

$$\frac{dU}{dR} = 1, \tag{6}$$

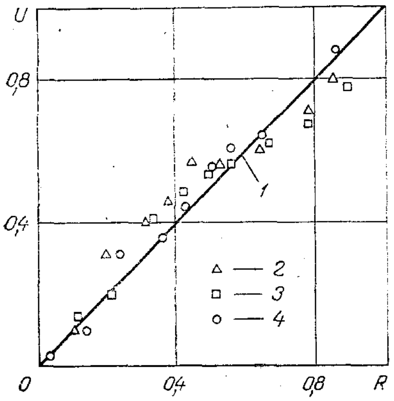
$$U = R, \tag{7}$$

$$\frac{d\Theta}{dR} = 1, \tag{8}$$

$$\Theta = R, \tag{9}$$

where $R = \ln(y/\delta_0)/\ln(\delta/\delta_0)$; $U = (u - u_0)/(u_\delta - u_0)$; $\Theta = (T - T_0)/T_\delta - T_0)$, $u_0 = u(x, \delta_0(x))$ $u_\delta = u(x, \delta(x))$; $T_0 = T(x, \delta_0(x))$; $T_\delta = T(x, \delta(x))$. Note that in the variables u^+, ϑ^+, y^+ we have $R = \ln(y^+/\delta_0^+)/\ln(\delta^+/\delta_0^+)$, $U = (u^+ - u_0^+)/(u_\delta^+ - u_0^+)$, $\Theta = (\vartheta^+ - \vartheta_0^+)/(\vartheta_\delta^+ - \vartheta_0^+)$.

Expressions (6)-(9) are rigorously defined in the interval $R \in [0, 1]$ if the boundary curve $\delta_0(x)$ directly separates the molecular and molar transfer zones in, for example, a turbulent boundary layer without a Karman transition zone.



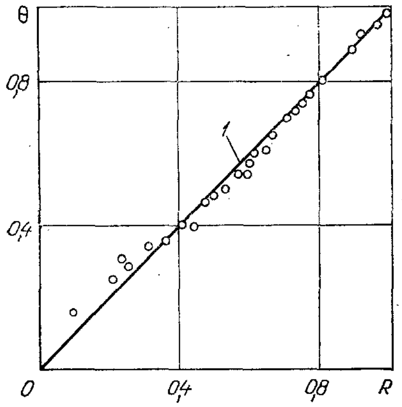


Fig. 1 Fig. 2

Fig. 1. Distribution of the generalized dimensionless velocity U(R) for $(\nu/u_\delta^2)(du_\delta/dx) = 9.42 \cdot 10^{-6}$ and various values of the average diameter d and volume concentration c of polystyrene beads. 1) Relation (7); 2) d = 0.32 mm, c = 0.2%; 3) 0.32 mm, 0.44%; 4) 0.82 mm, 0.2%.

Fig. 2. Distribution of the generalized dimensionless temperature Θ(R) for a turbulent water flow. 1) Relation (9).

In the case where the molar and molecular transfer regions are separated by a mixed-transfer transition region, the interval of R in which relations (6)-(9) are valid with sufficient accuracy is narrowed to $[R_1, 1]$, where $R_1 > 0$. The value of R_1 depends on the width of the transition zone $\delta_K(x)$, because $R_1 = \ln(\delta_K/\delta_0)/\ln(\delta/\delta_0)$. Here $\delta_0(x) < \delta_K(x) < \delta(x)$.

In Fig. 1, as an example, the universal relation (7) is compared with experimental data from an investigation [3] of a disperse turbulent flow with negative pressure gradients. The flow was made disperse by injecting spherical polystyrene beads of various diameters into it at various volume concentrations. Satisfactory correlation is obtained between the experimental data and expression (7).

In Fig. 2 the experimental data of [4] are compared with expression (9). The temperature profiles in the laminar sublayer and wall zone of the turbulent core were measured with high accuracy in [4] by means of a special microthermocouple. The satisfactory agreement of the experimental data with the theoretical relation (9) is obvious.

The molar transfer equations (6) and (8) are analogs of the molecular transfer equations (4) and (5), but (6) and (8) are formally unrelated to (4) and (5) and have greater universality. For example, it has been shown [2] that in molar transfer on a rough surface with degeneracy of Eq. (4), expression (6) is still valid. And in this case, rather than the thickness of the laminar sublayer as the limit of validity of Eq. (4), it is necessary in Eq. (6) to use the distance from the wall where the turbulent moles (eddies) "forget" the cause of their origin (roughness) and "behave" as if they literally originated at the sublayer boundary $\delta_0(x)$ specified by the relation $\ln\delta_0^+/(u_0^+ - 1) = 1$.

We rewrite Eq. (7), separating out the term that does not depend on the instantaneous values of y:

$$\Psi = \Psi_\delta, \tag{10}$$

where $\Psi = \ln(y^+/\delta_0^+)/(u^+ - u_0^+)$, $\Psi_\delta = \ln(\delta^+/\delta_0^+)/(u_\delta^+ - u_0^+)$. Substituting the value of Ψ into (10), we obtain

$$u^+ = \frac{1}{\Psi_\delta} \ln(y^+/\delta_0^+) + u_0^+. \tag{11}$$

From the integral of Eq. (1), which has been shown [2] to be valid for a zero pressure gradient, we obtain analogously

$$\tilde{\Psi} = \tilde{\Psi}_\delta, \tag{12}$$

$$u^+ = \frac{1}{\tilde{\Psi}_\delta} \ln y^+ + 1, \tag{13}$$

where $\tilde{\Psi} = \ln y^+/(u^+ - 1)$, $\tilde{\Psi}_\delta = \ln\delta^+/(u_\delta^+ - 1)$.

For a zero pressure gradient

$$\Psi_\delta = \tilde{\Psi}_\delta. \tag{14}$$

Thus, after simplifying expression (14) we obtain $\tilde{\Psi}_\delta = \ln\delta_0^+/(u_0^+ - 1)$, which corresponds to Eq. (12). Therefore, Eq. (1) is a special case of Eq. (6) and is valid under condition (14).

By analogy with expressions (10) and (11), we write Eq. (9) in the form

$$X = X_\delta, \tag{15}$$

$$\vartheta^+ = \frac{1}{X_\delta} \ln (y^+/\delta_0^+) + \vartheta_0^+, \tag{16}$$

where $X = \ln(y^+/\delta_0^+)/(\vartheta^+ - \vartheta_0^+)$, $X_\delta = \ln(\delta^+/\delta_0^+)/(\vartheta_\delta^+ - \vartheta_0^+)$.

A comparison of (11) and (16) shows that in the molar transfer domain, if the critical functions X_δ and Ψ_δ are equal, the profiles of the dimensionless temperature $\vartheta^+ - \vartheta_0^+$ and velocity $u^+ - u_0^+$ coincide:

$$\frac{\vartheta^+ - \vartheta_0^+}{u^+ - u_0^+} = \frac{\Psi_\delta}{X_\delta}. \tag{17}$$

We prove that in the first approximation

$$\frac{\Psi_\delta}{X_\delta} = 1. \tag{18}$$

Thus, inasmuch as $u_0^+ = \delta_0^+$ and, for $Pr = 1$, $\vartheta_0^+ = \delta_0^+$, an inspection of expression (17) reveals that in the case $\vartheta_\delta^+ = u_\delta^+$ Eq. (18) holds. The condition $\vartheta_\delta^+ = u_\delta^+$, on the other hand, is known as the quantitative form of representation of the Reynolds analogy [5].

It is evident from expression (5) that with an increase in Pr in the explicit form of the right-hand side of expression (16) only the value ϑ_0^+ read for the function ϑ^+ increases. Consequently, for $Pr > 1$ relation (18) is still valid in the first approximation.

In [6] the following equation has been deduced from Eq. (4) and its integral:

$$y^+ \frac{du^+}{dy^+} = u^+, \tag{19}$$

which the author interprets as follows: "Laminar flow can be represented by a superposition of eddies rolling along the wall with a dimensionless angular velocity du^+/dy^+ and with a translational velocity at point y^+ equal to the flow velocity u^+."

From the turbulent-core equations (6) and (7) we also formally deduce the rolling equation

$$R \frac{dU}{dR} = U, \tag{20}$$

analogous to (19). Here R is the generalized dimensionless distance from the boundary of the laminar sublayer δ_0, and dU/dR is the generalized dimensionless angular velocity.

Equation (20) describes the superposition of average turbulent eddies rolling along the boundary of the laminar sublayer. However, it can also be regarded as a superposition of average turbulent eddies, each of which rolls over its own surface situated at a distance y^+ from the wall and moving with the velocity given by expression (11). The rolling of the eddies is treated essentially as inherent in the reference system. Then at a dimensionless distance from the indicated surface l^+ equal to the dimensionless eddy radius, the eddy has a dimensionless rolling translational velocity u_l^+:

$$l^+ \omega_l^+ = u_l^+. \tag{21}$$

Here ω_l^+ is the dimensionless angular velocity of the average eddy.

Since the eddy superpositions given by Eqs. (20) and (21) must be equivalent, we make use of the fact that $l/y \leqslant 1$ to obtain from (11)

$$u_l^+ = u^+((y^+ + l^+)/\delta_0^+) - u^+(y^+/\delta_0^+) = \frac{l^+}{\Psi_\delta y^+}. \tag{22}$$

A comparison of expressions (22) and (21) shows that

$$\omega_l^+ = \frac{1}{\Psi_\delta y^+}. \tag{23}$$

From Eq. (11) we obtain

$$\frac{du^+}{dy^+} = \frac{1}{\Psi_\delta y^+}. \tag{24}$$

Now

$$\omega_l^+ = \frac{du^+}{dy^+} \tag{25}$$

and expression (21) acquires the form

$$l^+ \frac{du^+}{dy^+} = u_l^+. \tag{26}$$

Equation (26), unlike (20), is the physical as well as the formal analog of Eq. (19).

Inasmuch as the average eddy of width l^+ must completely transfer momentum flux density across the surface over which it is rolling, we have

$$u^+ = \sqrt{\tau^+}, \tag{27}$$

where $\tau^+(x, y) = \tau(x, y)/\tau_w(x)$. Then the expression for the transverse eddy dimension (radius) can be written in the form

$$l^+ = \Psi_\delta y^+ \sqrt{\tau^+}. \tag{28}$$

On the basis of condition (14) and expression (12),

$$l^+ = \tilde{\Psi}_\delta y^+ \sqrt{\tau^+} = \tilde{\Psi} y^+ \sqrt{\tau^+}. \tag{29}$$

As $y^+ \to 1$ the quantity $\tau^+ \to 1$ and, generalizing expression (29) to the laminar sublayer, we infer that

$$l^+ = \tilde{\Psi} y^+, \tag{30}$$

where automatically, taking the integral of Eq. (4) into account, we have $\tilde{\Psi} = \ln y^+/(y^+ - 1)$. In the laminar sublayer the function $\tilde{\Psi}$ decreases from $\tilde{\Psi}_* = 1$ at $y^+ = 1$ to $\tilde{\Psi}_0 = \tilde{\Psi}_\delta$ at $y^+ = \delta_0^+$.

Expression (30) explains the basic nature of the Kline streaks [7]. When local fluid masses (moles) become densely clustered near the wall in the laminar sublayer (where, marked with a dye, they form a streak), the radius of the mole in expression (30) and, hence, the structure of the streak change very slightly in expression (30) as the masses move away from the surface, because y^+ increases and, simultaneously, $\tilde{\Psi}$ decreases.

The moles in the laminar sublayer are ostensibly indistinguishable. However, when $\tilde{\Psi}$ attains a value $\tilde{\Psi}_0$ equal to $\tilde{\Psi}_\delta$, the scale l^+ just after the boundary of the laminar sublayer, according to (29), begins to grow rapidly, whereupon the local fluid masses in the turbulent core become individualized and the Kline streaks break up.

If the main flow is supersonic, the moles (eddies) emerging from the laminar sublayer are formed at the wall, where the Mach number $M_w = 0$, and so it follows from the assumption of their stability against dissolution during rolling along the boundary of the laminar sublayer that Eqs. (6) and (7) can be generalized to the case of molar transfer in supersonic flow.

As an example, Fig. 3 shows a plot of the integral of Eq. (1)

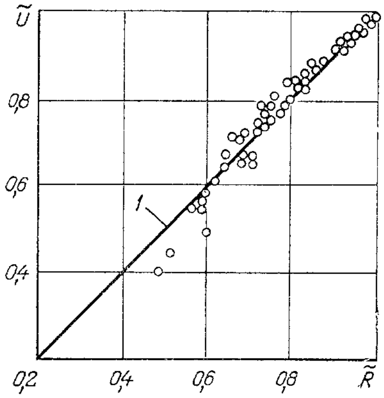


Fig. 3. Distribution of the generalized dimensionless velocity $\tilde{U}(\tilde{R})$ in supersonic turbulent flow. 1) Relation (31).

$$\tilde{U} = \tilde{R} \tag{31}$$

and the experimental data of an investigation [8] of the singular aspects of a turbulent boundary layer for a freestream Mach number of the order of 9. Satisfactory agreement is witnessed between the experimental data and the theoretical relation (31) beginning with $\tilde{R} \sim 0.6$–0.7. We note that the value $\tilde{R} \sim 0.6$ in the indicated experimental work corresponds to $y^+ \sim 10$, i.e., to the beginning of the turbulent core.

The generalized relations derived here for molar momentum and heat transfer in the turbulent core exhibit certain laws governing a turbulent boundary layer and can be used to simplify the calculations.

NOTATION

u, average longitudinal velocity, m/sec; T, average temperature, °K; T_w, wall temperature, °K; ν, kinematic viscosity coefficient, m²/sec; ρ, density, kg/m³; c_p, specific heat, J/kg•K; τ, tangential stress, N/m²; τ_w, tangential stress at wall, N/m²; q_w, specific heat flux at wall, W/m²; $u_* = \sqrt{\tau_w/\rho}$, dynamic velocity, m/sec; $\vartheta_* = q_w/\rho c_p u_*$, characteristic temperature, °K; δ, thickness of boundary layer, m; δ_0, thickness of laminar sublayer, m; $l_* = \nu/u_*$, transverse space scale of average mole at wall, m; $y^+ = y/l_*$, dimensionless coordinate; $u^+ = u/u_*$, dimensionless velocity; $\vartheta^+ = (T_w - T)/\vartheta_*$, dimensionless temperature; $\tau^+ = \tau/\tau_w$, dimensionless tangential stress; $R = \ln(y^+/\delta_0^+)/\ln(\delta^+/\delta_0^+)$, generalized dimensionless coordinate; $U = (u^+ - u_0^+)/(u_\delta^+ - u_0^+)$, generalized dimensionless velocity; Pr, Prandtl number. Indices: *, flow parameters evaluated at $y^+ = 1$; δ, parameters at $y^+ = \delta^+$; 0, parameters at $y^+ = \delta_0^+$; w, parameters at wall.

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